Charging kinetics of dust particles with a variable mass

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Abstarct

A kinetic equation for dust particles with a new kinetic variable - the mass of grains - is suggested for the description of processes with changing mass in dusty plasmas and neutral systems.

I. Introduction

The kinetic theory of dusty plasmas, which takes into account specific processes of charging, has been considered in many papers, but usually not from first principles. In [1] the dust charge was introduced as a new dynamic variable for the kinetic equation, in [2,3] collision integrals for dusty plasmas with charging have been formulated and used for several applications. The form of the charging collision integrals suggested in [2,3] has been recently rigorously justified in [4,5], where also the stationary velocity and charge distributions for dusty plasmas were established. In this report we consider the generalized kinetic equation in which a new dynamic variable - the mass of dust particles - is introduced in parallel with the charge variable. We will show that for models of dusty plasmas with absorption of ions and electrons the distribution function of grains and the average kinetic energy are determined not only by momentum transfer from light plasma particles to dust particles, but essentially (on the same time scale) also by mass transfer. This statement also agrees with the preliminary results of MD simulations of the heating of dust particles in plasma [6]. A simplified form of the obtained kinetic equation (the nonstationary variant of the Fokker-Planck equation with changing mass) is also found and the simplest concrete applications are considered. For some more complicated situations with surface chemical reactions between electrons and ions absorbed by dust particles, when atoms appear and can return from the dust to plasma, the processes of mass transfer can also be essential. It is necessary to emphasize that the formulated equation can be important for different applications not only for plasmas, but for other systems with mesoscopic particles, where processes with mass transfer take place.

II. Kinetic equation for dust particles with mass and charge variables

Let us introduce the generalized kinetic equation in which a new dynamic variable M - the mass of dust particles is included in parallel with the charge Q:

$$\frac{df_D(t)}{dt} = J_D(\vec{p}, \vec{r}, t, Q, M) + J_D^c, \tag{1}$$

where the collision integral J_D^c describes all collision processes without change of number of small particles (e.g. electrons and ions in plasmas) and without change of mass and charge of grains. The collision integral J_D describes the absorption of mass and charge by dust particles. For the simple model of absorption of electrons and ions with masses m_α and charges e_α by a grain with charge Q we can write:

$$J_D = \sum_{\alpha=(e,i)} \int d\vec{p} f_\alpha(\vec{p},\vec{r},t)\big(\omega_\alpha(\vec{p},\vec{P}-\vec{p},Q-e_\alpha,M-m_\alpha) f_D(\vec{P}-\vec{p},\vec{r},t,Q-e_\alpha,M-m_\alpha) \\ -\omega_\alpha(\vec{p},\vec{P},Q,M) f_D(\vec{P},\vec{r},t,Q,M)\big). \quad (2)$$

where $\omega_\alpha(\vec{p},\vec{P},Q,M) = v\sigma_\alpha(v,Q,M)$ is the probability density of absorption of an electron or ion with momentum $\vec{p}$ and charge e_α by a grain with momentum $\vec{P}$ and charge Q. This collision integral implies that the processes of mass transfer from grains back to the plasma are absent. If there are such type of processes the more complicated equations can be written to take into account the mass balance in plasmas correctly. The simplest approximation for the cross-section of absorption σ_α can be chosen in the usual form (see e.g. [3]). In general the cross-section can be some function of the mass of grains (for example due to the dependence of the grain's radius on mass). In general the necessity to include the mass as a new kinetic variable depends, naturally, on the time scale, under consideration. To simplify the problem we have to expand the collision integrals using the small parameters $\frac{m_\alpha}{M}$ and $\frac{e_\alpha}{Q}$. We also suggest here that $P \gg p$. Then we can find the generalized Fokker-Planck equation for grains, which will be nonstationary in our case, due in particular to mass absorption. In this paper we will realize this expansion for a neutral system: neutral grains in system of small neutral particles. Simplification of Eq.1 and Eq.2 for plasmas is similar and will be published separately.

III. Kinetics of neutral grains due to mass absorption collisions

For neutral homogeneous systems we can rewrite Eq.2 in the form

$$J_D(P,M,t) = \int d\vec{p} f_n(\vec{p},t)\big(w(\vec{p},\vec{P}-\vec{p},M-m) f_D(\vec{P}-\vec{p},M-m,t) \\ -w(p,P,M) f_D(P,M,t)\big). \quad (3)$$

Here $w(p,P,M) = \sigma(M)|\frac{\vec{P}}{M} - \frac{\vec{p}}{m}|$.

After expansion up to first order in $\frac{m_\alpha}{M}$ and up to second order in $\frac{p}{P}$ we find the kinetic equation for grains:

$$\frac{\partial f_D}{\partial t} = \frac{j_\epsilon \sigma(M)}{3} \Delta_P f_D(P,M,t) + \frac{j_0 m \sigma(M)}{3M} \frac{\partial}{\partial P_\alpha}[P_\alpha f_D(P,M,t)] \\ -j_0 m \frac{\partial}{\partial M}[\sigma(M) f_D(P,M,t)] , \quad (4)$$

where:

$$j_0 = \int d\vec{p} \frac{p}{m} f_n(p,t), \qquad j_\epsilon = \int d\vec{p} \frac{p^3}{2m} f_n(p,t). \quad (5)$$

Below we will suggest that $f_n(p,t) = f_n(p)$ is stationary Maxwell distribution for small particles with temperature T_0 and density n_0. Then we find:

$$j_0 = \frac{4 n_0 T_0^{1/2}}{\sqrt{2\pi m}}, \qquad j_\epsilon = 2 m T_0 j_0. \quad (6)$$

Let us consider some average functions: density n_D, mass of grain $< M >$ and $U_{ab}(\lambda) =< \frac{p^a}{\lambda M^b} >$, where we define the averaging as

$$n_D = \frac{1}{M_0}\int d\vec{P}dM f_D(P,M,t), \qquad < A >= \frac{1}{n_D M_0}\int d\vec{P}dM f_D(P,M,t)A \quad . \tag{7}$$

Then we find:

$$\frac{dn_D}{dt} = 0, \qquad \frac{d < M >}{dt} = \frac{j_0}{n_D}\int d\vec{P}dM\sigma(M)f_D(P,M,t) > 0 \quad , \tag{8}$$

$$\frac{dU_{ab}(\lambda)}{dt} = -\frac{mj_0(a+3b)}{3M_0 n_D}\int d\vec{P}dM\frac{P^a}{\lambda M^{b+1}}\sigma(M)f_D(P,M,t) + \frac{j_\epsilon a(a+1)}{3M_0 n_D}\int d\vec{P}dM\frac{P^{(a-2)}}{\lambda M^b}\sigma(M)f_D(P,M,t) \quad . \tag{9}$$

The question arises whether stationary averages in the limit $t \to \infty$ are possible. For example for the average kinetic energy of grains $E(t) = U_{21}(2)$ we find, as follows from Eq.9, the stationary solution in the limit $t \to \infty$ in the case $\sigma(M) \sim M$ ($\sigma \equiv \sigma_0' M$):

$$\lim_{t\to\infty} < E(t) >= \frac{6}{5}T_0 \quad . \tag{10}$$

We emphasize that if we formally omit the last term in Eq.4, describing change of the mass M, the stationary Maxwell distribution function for grains with the temperature $T_D = 2T_0$ can be immediately found. This result coincides with the solution obtained in [4] for the limit of uncharged particles. Really the omitted term is of the same order as other terms in Eq.4, as shown above. Nevertheless the physical results and predictions obtained in [4] can be realized if the physical process of transfer of atoms from the surface of dust particles to the plasma take a place and is included in the kinetic theory. In this case the mass of dust particles can be fixed due to this process. A more detailed analysis of the problem of stationary averages in the limit $t \to \infty$ for neutral systems and dusty plasmas in parallel with the consideration of the nonstationary solutions of Eq.4 for different cases, in particular, of such as (for the case $\sigma(M)$ =const):

$$f_D(P,M,t) = \varphi(t - \frac{M}{mj_0\sigma})\ \chi(P,M) \tag{11}$$

and some others, including the solutions of generalized Fokker-Planck equation for dusty plasmas, will be presented separately.

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